The Shaft Modeling Tool Kit
(A Software Package for the
Static and Dynamic Analysis of Marine
Shaftlines)

by

D.N. Bruce Cowper
and
Alan Murray

PRESENTED AT THE
SNAME PACIFIC NORTHWEST SECTION MAY 1989

Fleet Technology Limited
16, 6325 11th Street S.E.
Calgary, Alberta T2H 2L6
SUMMARY

A number of successful full scale tests of icebreaker propulsion systems have been completed in recent years. The principal motivation for much of this recent work has been to provide an adequate experimental basis for the development and validation of numerical modeling techniques for predicting the performance of alternative shafting and bearing arrangements. Such models should not only provide appropriate quantitative information concerning the loads experienced by key system components such as shaft bearings, but should also reproduce the overall qualitative behavior of the system observed at full scale. The main objective of the work reported in this paper was to develop a software package to access the full scale data and execute numerical models for the design and evaluation of ice class shafting and propeller systems.

A software package, called the Shaft Modeling Tool Kit, was developed for this purpose. It is the first available comprehensive software package for the analysis of ice class shafting and propeller systems. The suite of programs (written for IBM PC/AT compatibles) in the Shaft Modeling Tool Kit provide the following:

- Access to key Full Scale Data from several ice class vessels
- Tabular Displays of the Canadian Arctic Shipping Pollution Prevention Regulations (CASPPR) for Powering, Shafting, and Propellers
- Analytical Modeling of generalized shafting and propeller systems for Torsion, Thrust, and Alignment (Dynamic and Static)

The program is entirely menu-driven, that is, the user selects the program function to be performed from lists, or menus. Items are selected from menus using a mouse. By moving the mouse, the user places an arrow on the monitor over the item in the menu he wishes to select and then presses the mouse button.

The program is currently being used by designers of ice class shafting and propeller systems. It has proven to be an extremely useful design tool by providing the user with the calculated dynamic response of an ice class shafting and propeller system in a matter of minutes. High quality graphic functions enable the user to evaluate the calculated response of the model by comparing the time history results to measured full scale data.
1. INTRODUCTION

During the past fifteen years Canada has achieved a position of world leadership in many aspects of the technology of arctic marine transportation. This has come about both through the efforts of private industry and through the systematic technical programs of Government of Canada agencies such as the Canadian Coast Guard and the Transportation Development Centre of Transport Canada. The private companies involved have been concerned primarily with the technical and economic problems posed by the transportation and operational requirements of Arctic resource development. Government agencies have been primarily concerned with providing the regulatory framework for industrial operations in the Arctic and with the problems of operating and maintaining a growing fleet of ice capable shipping employed in carrying out Government missions.

A prominent feature of the Canadian approach in this field has been the emphasis placed upon the acquisition of full scale engineering data and the use of operational experience to guide and inform the technology development process. This is natural considering that, in Canada, our environment presents a particularly wide range of sea ice conditions and we have had the opportunity to operate a wide range of differing ship sizes and designs in these conditions. Since the early seventies Canadian agencies and companies have either conducted or participated in more than forty major programmes of full scale experimentation with ice going ships.

The reliability of the propulsion system is one of the most important considerations in the design and operation of ice going shipping, for two reasons. Firstly, the propellers, shafting, shaft seals and shaft bearings are amongst the most vulnerable parts of an ice transiting ship. They are exposed to repeated, very high amplitude loads and vibration levels. Secondly, the failure of this equipment has a great impact on the ability of the ship to perform its mission and hence upon the overall economics of its operation.

The design practice for ice-going propulsion machinery has evolved much the same way as the design criteria for ice breaking hulls; experience of damage and failure of system components in early designs has led to progressive increases in strength requirements, typically expressed as an ice strengthening factor applied to the strength requirements for clear water designs. Over the past several decades this approach has been successful in substantially controlling the incidence of catastrophic failure and in reducing progressive deterioration of system performance to manageable levels.
However, the large body of empirical design rules developed in response to early system failures does not necessarily provide adequate guidance for the specification of entirely new types of propulsors which take advantage of new technology. Such developments as ice going controllable pitch propellers, ducted propellers for enhanced thrust and new types of prime movers with different control and torque delivery characteristics all offer attractive advantages to the system designer, but may give rise to entirely new problems not anticipated in classical design practice. What is needed in such circumstances is a reliable, qualitative model of how of ice-propeller interaction loads arise and are propagated in a generalized propulsion system.

The principal motivation for much of this recent work has been to provide an adequate experimental basis for the development and validation of numerical modeling techniques for predicting the performance of alternative shafting and bearing arrangements. This paper describes the development of a comprehensive software package (written for IBM PC/AT compatibles) for the analysis of ice class shafting and propeller systems. The suite of programs in this package, called the Shaft Modelling Tool Kit) provide the following:

- Access to key **Full Scale Data** from several ice class vessels dedicated tests
- Tabular Displays of the **CASPPR Regulations** for Powering, Shafting, and Propellers
- Analytical **Modeling** of generalized shafting and propeller systems for Torsion, Thrust, and Alignment (Dynamic and Static)
2.0 Program Structure

The program is entirely menu-driven, that is, the user selects the program function to be performed from lists, or menus. The menus are organized hierarchically, such that selecting an item from one menu may enable other menus. Items are selected from menus with a "mouse". Figure 2.1 illustrates the overall program structure flow chart. The following sections describe the program organization of each of the three main sections of the program.

Database

Figure 2.2 illustrates the structure of the database section of the program which provides quick access to data pertaining to several ice-class vessels, including component specifications and data from full scale propulsion system monitoring. The full scale data included was selected after an extensive review of the many documents produced and data collected from tests conducted between 1977 and 1987. The data selected represents typical and extreme events for each vessel, including ice-propeller interaction such as milling, blockage in the case of nozzled propellers, and short duration impacts. Data is presented in tables and time history plots. The user of the program can view the specifications of vessel components; display a synopsis of the full scale tests including the environmental conditions and test specifics; and view the time history of any signal collected during any one of the tests.

Modeling

Figure 2.3 illustrates the structure of the modeling section of the program. Tables 2.1 and 2.2 list the model input and output parameters respectively. The modeling section of the program provides the user with a tool to predict the dynamic response of an arbitrary shafting system due to ice-propeller interaction. Analysis options are available to calculate Torsional (Torque Model), Axial (Thrust Model), and Transverse (Alignment) dynamic response. A static alignment analysis option is also provided. Components such as the propeller, shafting, flywheel, oil distribution box and gearbox can be changed and specified by the user. Ice forces on the propeller used in the calculations can be specified or calculated using state of the art ice-propeller interaction algorithms.

Regulations

The regulations section of the program provides the user with a description of the current CASPPR pertaining to the powering and machinery requirements. This data is useful for comparing data in the full scale data base and results obtained from modeling with the expected loads and component specifications as described in the regulations.
Figure 2.2    Structure of Data Base Section
Figure 2.3 Structure of Modeling Section
<table>
<thead>
<tr>
<th>Table 2.1</th>
<th>Model Input Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shaft Elements</strong></td>
<td>Units</td>
</tr>
<tr>
<td>Outside Diameter</td>
<td>meters</td>
</tr>
<tr>
<td>Inside Diameter</td>
<td>meters</td>
</tr>
<tr>
<td>Length</td>
<td>meters</td>
</tr>
<tr>
<td>Modulus of Elasticity (E)</td>
<td>GPa.</td>
</tr>
<tr>
<td>Torsional Modulus</td>
<td>GPa.</td>
</tr>
<tr>
<td>Mass Density</td>
<td>kg/m³</td>
</tr>
<tr>
<td><strong>Propeller</strong></td>
<td>Units</td>
</tr>
<tr>
<td>Type</td>
<td>1: Nozzled, 2: Open</td>
</tr>
<tr>
<td>Diameter</td>
<td>meters</td>
</tr>
<tr>
<td>Hub Diameter</td>
<td>meters</td>
</tr>
<tr>
<td>Expanded Area Ratio</td>
<td>/</td>
</tr>
<tr>
<td>RPM</td>
<td>/</td>
</tr>
<tr>
<td>Blade Width</td>
<td>meters</td>
</tr>
<tr>
<td>Blade Tip Thickness</td>
<td>mm</td>
</tr>
<tr>
<td>Blade Pitch</td>
<td>degrees</td>
</tr>
<tr>
<td>Polar Moment of Inertia</td>
<td>kgm²,</td>
</tr>
<tr>
<td>Mass</td>
<td>kg</td>
</tr>
<tr>
<td>Damping</td>
<td>Torsion: kgm²/sec, Thrust: kg/sec, Alignment: kg/sec</td>
</tr>
<tr>
<td>Number of Blades</td>
<td>/</td>
</tr>
<tr>
<td>Blade Root Thickness</td>
<td>mm</td>
</tr>
<tr>
<td>CASPPR Class</td>
<td>/</td>
</tr>
<tr>
<td>Ship Speed</td>
<td>m/sec</td>
</tr>
<tr>
<td><strong>Bearings</strong></td>
<td>Units</td>
</tr>
<tr>
<td>Type</td>
<td>1: Thrust, 2: Vertical, 3: Torsion, 4: Bending</td>
</tr>
<tr>
<td>Stiffness</td>
<td>Torsion: Nm/rad, Thrust: N/m, Bending: Nm/rad, Vertical: N/m</td>
</tr>
<tr>
<td>Offset</td>
<td>mm (Only Static Alignment Analysis, Vertical Bearing)</td>
</tr>
<tr>
<td><strong>Concentrated Masses and Inertias</strong></td>
<td>Units (for all Types)</td>
</tr>
<tr>
<td>Electric Motors</td>
<td>Torsional Inertia: kgm², Mass: kg</td>
</tr>
<tr>
<td>Gear Wheel</td>
<td></td>
</tr>
<tr>
<td>Flywheel</td>
<td></td>
</tr>
<tr>
<td>Couplings</td>
<td></td>
</tr>
<tr>
<td>Oil Distribution Boxes</td>
<td></td>
</tr>
<tr>
<td><strong>Integration Parameters</strong></td>
<td>Units</td>
</tr>
<tr>
<td>Time</td>
<td>seconds</td>
</tr>
<tr>
<td>Time Step</td>
<td>seconds</td>
</tr>
<tr>
<td>Damping Coefficient Alpha</td>
<td>/</td>
</tr>
<tr>
<td>Damping Coefficient Beta</td>
<td>/</td>
</tr>
</tbody>
</table>
### Table 2.2  Model Output Parameters

<table>
<thead>
<tr>
<th>Output Parameter</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque at each beam element</td>
<td>Torque Model</td>
</tr>
<tr>
<td>Thrust at each beam element</td>
<td>Thrust Model</td>
</tr>
<tr>
<td>Reaction of each bearing</td>
<td>All Models</td>
</tr>
<tr>
<td>Displacement at each node</td>
<td>Dynamic Alignment Model</td>
</tr>
<tr>
<td>Propeller Force Time History</td>
<td>All Models</td>
</tr>
</tbody>
</table>

### 3.0 Analysis Procedure/Methodology

### 3.1 Structure of Shafting Model

The development of an effective model for determining the dynamic response of ice class shafting systems requires a knowledge of the following:

1. Full scale data
2. Structural dynamics
3. Numerical analysis
4. Computer applications

Full scale data provides qualitative and quantitative information on the dynamic response of real systems, such as: measured shaft torque and thrust, bearing stiffness and damping. The development of accurate and efficient structural elements requires a background in structural dynamics. The efficiency of a computer modeling program depends largely on the numerical techniques employed and on their effective computer implementation. With regard to programming techniques, an optimum allocation of high and low speed storage is necessary.
The developed shaft model is composed of a linear system of continuous beam elements with concentrated springs (bearings), damping (propeller damping), and masses (electric motors, flywheels, etc.). An illustration of the M.V. Kalvik shaftline along with the model representation is shown in Figure 3.1. This type of modeling representation of the shafting allows for parametric variation of model components in terms familiar to marine engineers. Each beam element has a node at both ends with up to six degrees of freedom per node. Concentrated springs (bearings), torsional inertias, and masses can be applied at any node in the model. Ice forces on the propeller are included by applying an input force waveform on the node representing the propeller (extreme left node).

Besides a natural frequency determination for the system, three types of dynamic response calculations are included in the analysis: Thrust, Torque, and Alignment. The shafting model must be composed of at least one shaft, one bearing, and a propeller, and the type and number of bearings must be chosen such that rigid body motion is inhibited.

3.2 The Solution Technique

The equilibrium equations of motion for a linear system of structural elements, such as a shaftline model, yield a set of linear equations of the following form:

\[
[M] \ddot{y} + [C] \dot{y} + [k] y = [F]
\]

where:

- \([M]\) - Mass Matrix of beam element
- \([C]\) - Damping Matrix of beam element
- \([k]\) - Stiffness Matrix of beam element
- \([F]\) - Forcing Function Matrix (Ice Forces on Propeller)
- \(\ddot{y}\) - nodal acceleration matrix
- \(\dot{y}\) - nodal velocity matrix
- \(y\) - nodal displacement matrix
Figure 3.1 Modeling of the M.V. Kalvik Shaft Line
The structural matrices (for the entire model) are formed by direct addition of the individual element matrices: for example

\[ [k] = \sum [k_m] \]

where \( k_m \) is the stiffness matrix of the \( m \)th element. Although \( k_m \) is formally of the same order as \( k \), only those terms in \( k_m \) which pertain to the element degrees of freedom are nonzero. The addition of the element matrices can therefore be performed by using the element matrices in compact form together with identification arrays which relate local element to global structure degrees of freedom.

Based upon previous modeling experience and full scale data analysis, it was determined that the preferred modeling solution technique must conform to the following criteria:

1. Fast calculation time
2. Proven reliability
3. Ability to determine system response for complex input forcing functions (ice force on the propeller)
5. Ability to expand to more complex modeling for future development, without extensive modification to software code. Additions may include:
   - ice interaction algorithm (input force is dependent upon the response of the shaftline)
   - non-linear spring elements
6. Solve response in the form of a time history
7. Unconditionally stable

The two most common solution techniques for structural dynamic problems are those of modal superposition and the direct integration method. In a conservative system each of the natural mode shapes are unique and this property, known as the Principal of Orthogonality, may be used to uncouple the equations of motion and hence determine the system response to a given forcing function. Strictly speaking when an elastic structure experiences a sudden disturbing force it responds dynamically, all of its modes being excited to a greater or lesser extent. Practically speaking almost all of the system response is contained in the lower modes. Thus it is possible to closely approximate a system response in terms of a linear combination of its first few mode shapes. This procedure is known as mode superposition.
Direct integration is equivalent to a mode superposition analysis in which all the eigenvalues and vectors have been calculated and the uncoupled equations are integrated with a common time step $\Delta t$. The integration can only be accurate for those modes for which $\Delta t$ is smaller than a certain fraction of the period $T$ (inverse of frequency), usually 0.01 times $T$. Using the Wilson-$\theta$ method the integration errors effectively "filter" out of the solution the high mode response (ie. for which $\Delta t/T$ is large). This filtering is due to amplitude decay observed in the numerical solution when $\Delta t/T$ is large. The effective filtering of these high frequency responses from the solution may be beneficial. Integration accuracy cannot be obtained in the response of those modes for which $\Delta t/T$ is large and the filtering process allows one to obtain a total system solution in which the lower mode response is accurately observed. It is therefore noted that the direct integration technique is equivalent to a mode superposition analysis, in which only the lowest modes of the system, but a sufficient number to take proper account of the applied loading, are considered. The exact number of modes effectively included in the analysis depends on the time step size $\Delta t$ and the distribution of the vibration periods.

The advantages of the direct integration method over the mode superposition method is that it is more effective when the response is required over relatively few time steps, such as in shock problems (propeller-ice impacts). Also the direct integration technique can accommodate non-linear spring elements and the ability to develop ice-interaction algorithms, without extensive software code revision. Therefore, the Wilson-$\theta$ method of direct integration was chosen as it satisfied all of the criteria specified.

The Wilson-$\theta$ method is essentially an extension of the linear acceleration method in which the acceleration during any one time step is assumed to vary linearly over a range of $\theta$ times the solution time step. Paz, 1985 developed the equations of motion using the Wilson-$\theta$ method by adopting an incremental load approach. The original development by Wilson accounted for the total load at each step, but both methods give the same final equation.

Returning to equation 1, the equilibrium equations of motion:

$$[M] \ddot{y} + [C] \dot{y} + [k] y = [F]$$

These equations can be solved directly by determining the acceleration, velocity, and displacement, at successive time intervals in terms of the values at the preceding time $t$. Thus one can 'step through' a given solution, evaluating the system response at each successive time interval.
3.3 Determination of the System Natural Frequencies.

The limited nodal connections within any shaftline system results in stiffness and mass matrices which are narrow banded. A linear elastic dynamic system gives rise to equations of motion which take the form of a classical eigenvalue problem, the roots or eigenvalues of which are proportional to the natural frequencies of the shaftline. Interest is usually confined to the lower modes of vibration since these require least amounts of energy to excite. Taking account of both of these features an eigenvalue solution routine was incorporated within the package which permits up to ten natural frequencies to be calculated simultaneously.

In general where only a few frequencies are required, or when good estimates of the mode shapes are available either from previous tests or similar shaftlines, iteration methods are an efficient means of determining system natural frequencies. The best known of these methods is Von Mises' power iteration where a single trial mode shape vector is continually premultiplied by the system matrix until successive iterates become proportional to each other. This process yields the highest eigenvalue of the system matrix which corresponds to the lowest natural frequency. An extension of this method (Jennings 1967) involves iterating simultaneously with a number of trial mode shape vectors. The use of a simultaneous or sub space iteration solution approach is beneficial from a computing requirement viewpoint since a number of the system frequencies in any given shaftline are grouped relatively close together. This close spacing could significantly slow down the convergence process if the simpler single vector Power Iteration approach were to be used. Instead by operating simultaneously with a number of vectors each distinctive mode shape, even those associated with equal frequencies, is quickly uncoupled and convergence readily achieved.

In the case of both axial and torsional vibration the shaftline is capable of describing rigid body motions which, since they involve no straining of the shaftline would result in trivial, zero frequencies. These rigid body motions can be suppressed physically by affixing a light spring to both an axial and a torsional degree of freedom. Since there is no mass ascribed to such a spring its natural frequency is extremely high, well beyond the frequency range of interest. Finding the system natural frequencies in the above fashion serves as a useful check on the accuracy of the integration technique.
3.4 Damping

Relatively little is known about the evaluation of the damping coefficients which make up the damping matrix "C". It is convenient in the analysis of ice class shafting systems to represent the damping in the following manner:

\[ C = \alpha[M] + \beta[K] \]

where: \( \alpha \) and \( \beta \) are constants

The constants \( \alpha \) and \( \beta \) can be chosen to best fit the damping characteristics of the system. Typically a sensitivity analysis is done to determine the effect of changes in \( \alpha \) and \( \beta \) on the response of the shafting system. This is known as Rayleigh damping.

Using Rayleigh damping the modal damping ratio becomes: \( \xi_i = \alpha/(2\omega_i) + \beta\omega_i/2 \)

where: \( \xi_i = \) modal damping ratio  
\( \omega_i = \) natural frequency of the \( i \)th modal response

If \( \alpha \) is set to zero (0) then the damping becomes purely structural and \( \beta = 2\xi_i/\omega_i \)

for critical damping: \( \xi_i = 1 \)
\( \beta = 2/\omega_i \)

Typically the coefficients \( \alpha \) and \( \beta \) are determined empirically by the examination of full scale data.
3.5 Static Alignment

The evaluation of static alignment is accomplished by determining the displaced position of the shaft line under the effect of its own weight and specified offsets at the bearing locations.

The vertical bearing reactions for a shaft model are solved using the static equilibrium matrix equation:

\[
[F] = [k][y]
\]

where:

- \([F]\) = Force matrix
- \([k]\) = Stiffness matrix
- \([y]\) = Nodal displacement matrix (displacement of each node)

The static alignment algorithm performs the following:

1. Calculates the stiffness matrix as is done in the dynamic analysis
2. Calculates the force matrix based upon:
   - the shaft weight
   - the force required to move the shaft to the specified offset at each bearing
3. Solves the nodal displacement matrix for the calculated force and stiffness matrices
4. Calculates the vertical force, \(F_v\), in each shaft element, by multiplying the calculated nodal displacement by the appropriate stiffness at each node.
5. Calculates the bearing reactions from the nodal static equilibrium conditions (\(\sum F_v = 0\)).

Since the bearings can have appreciable length an accurate representation of their support function can be determined by refining the shaft model at each bearing location, by increasing the number of nodes and hence the number of nodal reactions. This facility improves upon current practice adopted by the Classification Societies in which a single point of support is assumed and its location is decided upon in a somewhat arbitrary fashion.

Full scale measurements conducted on the M.V. Kalvik in the fall of 1986 provided data on the load distribution throughout the aft stern tube bearing during open water and ice-propeller milling conditions. This data was used to validate the modeling of the bearing supports by a number of support springs.
4.0 Examples of Shaftline Analysis

Detailed shaftline vibrational and static alignment analysis have been done for several ice class and open water vessels using the Shaft Modeling Tool Kit. Each analysis is a report in itself; however the following is a brief summary of the results from the analysis of two ice class shaftlines. The static alignment analysis is for the shaftline of the MV Kalvik (an CASPPR Ice Class IV Vessel). The axial, lateral, and torsional vibration analysis is for a CASPPR Ice Class VIII shaftline.

Static Alignment Analysis

Figure 4.1 illustrates a model of the shaftline of the MV Kalvik. Figure 4.2 illustrates the results of the static alignment calculation in terms of bearing reaction magnitudes and Figure 4.3 shows the calculated displacement of the shaftline with the aft stern tube bearing displaced downward 5 mm. The user can change the bearing locations and offsets, then re-calculate the bearing reactions and displacement in less than one minute for the most complex of shaftlines.

Vibration Analysis

Figure 4.4 illustrates the model of an CASPPR Ice Class VIII shaftline. The following sections describe the dynamic analysis of this shaftline.

Axial Vibration Analysis

The objective of this analysis is to calculate the resonant axial vibration conditions of the shafting system. These were computed by the dynamic analysis of the beam element model of the shaftline, to determine the natural axial vibration frequencies. These frequencies were compared to the possible sources of excitation to determine the critical shaft speeds that could cause resonance.

Table 4.1 lists the calculated natural frequencies for the first mode of vibration of the shafting system for a series of thrust block stiffnesses. This mode is associated with the tail shaft, intermediate shaft, and propeller vibrating in unison against the thrust bearing.
Figure 4.1  Alignment Model of MV Kalvik Shaftline
Figure 4.2 Static Bearing Reactions of MV Kalvik Shaftline
Figure 4.3 Static Shaft Deflections of MV Kalvik Shaftline
Diagram(s) for Model: Example

Date/Time of Last Modification: 12/02/1989 12:38:16

Figure 4.4 Model of CASPPR Ice Class VIII Shaftline
Table 4.1  Axial Natural Frequencies
(First Mode)

<table>
<thead>
<tr>
<th>Stiffness (GN/m)</th>
<th>Natural Frequency (CPM) Centre Line Shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>480</td>
</tr>
<tr>
<td>2</td>
<td>630</td>
</tr>
<tr>
<td>3</td>
<td>710</td>
</tr>
</tbody>
</table>

The excitation frequency considered was that excited at the blade rate of 420 CPM for a service shaft speed of 105 RPM. Full scale measurements indicate that the axial excitation frequency during ice-propeller interaction is at the blade rate and in the range of 100-300% of the nominal thrust. The values in Table 4.2 indicate that a thrust bearing axial stiffness of 2 GN/m (200 tonnes/mm) or greater will ensure that the first mode axial natural frequency will be higher than the propeller blade rate; therefore, resonant axial vibration will not occur.

Lateral Vibration Analysis

This analysis was conducted to identify the resonant lateral vibration conditions of the shafting system. These were computed by the dynamic analysis of the beam element model of the shaftline, to determine their natural lateral vibration frequencies. These frequencies are compared to the possible sources of excitation to determine the critical shaft speeds that could cause resonance. The excitation frequency considered was that at the blade rate of 420 CPM for a service shaft speed of 105 RPM.

Predicted lateral vibration resonant speeds are dependent primarily upon the transverse flexibility (lateral stiffness) of the aft stern tube bearing, and the assumed point of support within this bearing. Lateral resonant speeds were therefore calculated for the flexibility range from 0.8 to 1.0 GN/m for the aft stern tube bearing, and for a support point of 0.2L and 0.3L from the aft end of the bearing. All other bearings were specified to have a stiffness of 1 GN/m (100 tonnes/mm). The calculated natural frequencies in RPM referred to shaft rate (critical speeds), due to propeller blade order excitations are listed in Table 4.2. Calculations indicated that the first mode of lateral vibration mode is associated with the cantilever vibration of the propeller and tail shaft. Since the calculated lateral natural frequencies were found to be above a 10% band of the maximum operating RPM (105 ± 10.5), the system will not experience lateral vibration in service.
Table 4.2  Lateral Natural Frequencies

<table>
<thead>
<tr>
<th>Support Point (Distance from aft end of aft stern tube bearing)</th>
<th>Stiffness (GN/m)</th>
<th>Natural Frequency (Referred to Shaft Rate) (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2L</td>
<td>0.8</td>
<td>130</td>
</tr>
<tr>
<td>0.2L</td>
<td>1.0</td>
<td>139</td>
</tr>
<tr>
<td>0.3L</td>
<td>0.8</td>
<td>120</td>
</tr>
<tr>
<td>0.3L</td>
<td>1.0</td>
<td>128</td>
</tr>
</tbody>
</table>

Note: L = Length of bearing

The program allows the user to view the bearing reactions in animated form to illustrate the distribution of bearing reactions for a series of time steps. It should be noted that current practices are to model the support of the aft stern tube bearing as a single support. The location of this point support affects the lateral vibration frequency as well as the bearing reaction distribution. Based upon the analysis of full scale data this is not an accurate representation for the dynamic response. In fact during ice-propeller interaction the bearing is loaded over a portion of it length and the location of the centroid of this support varies with time. This type of bearing load distribution causes the natural frequency to vary during ice-propeller interaction and also increases the effective damping of lateral vibration. The program can model this behavior and is a subject for future projects to improve the modeling techniques for lateral vibration of shaftlines.

Torsional Vibration Analysis

The objective of this study was to identify the resonant torsional vibrations of the shafting system. These were computed by the dynamic analysis of the beam element model of the shaft line, to determine the natural torsional vibration frequencies. These frequencies were compared to the possible sources of excitation to determine the critical shaft speeds that could cause resonance. Therefore, the investigation involved only the significant excitation forces and presents the natural frequencies that occur in the corresponding speed range. The excitation frequencies considered were those excited at the shaft service speed (60 - 105 RPM), and the blade rate (240 - 420 CPM).

The gear box was modeled as a lumped parameter system with the main gear wheel, and pinions as one lumped inertia connected to the clutch and fluid coupling through a flexible shaft. The fluid coupling was modeled as two lumped inertias separated by a flexible element with the manufacturer's specified stiffness and torsional inertias. The engine was modeled a series of lumped inertias as specified by the engine manufacturer. The engine flywheel and flexible
coupling were included in the system model as distinct elements. Table 4.3 lists the computed torsional natural frequencies.

**Table 4.3  Torsional Natural Frequencies (CPM)**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Zero Pitch</th>
<th>Full Pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>295</td>
<td>291</td>
</tr>
<tr>
<td>3</td>
<td>340</td>
<td>322</td>
</tr>
</tbody>
</table>

The first mode shape for cases 1 and 2 is associated the engine flexible coupling stiffness. The corresponding second mode shape occurs when the propeller vibrates against the gear box components. The engine flexible coupling isolates the torsional vibrations from the propeller to the main engines. The excitation frequencies considered were those induced at the shaft RPM and the propeller blade rate. The following excitation frequencies were considered:

- Shaft Rotation  
  - 60 - 105 CPM
- Blade Rate  
  - 240 - 420 CPM

Calculations were carried out to assess the significance of the coincidence of these excitations with the calculated natural frequencies. The results from a forced-damped response analysis indicated that torsional vibratory stresses corresponding to the calculated natural frequencies, when excited at the their respective resonant shaft speeds, were found to be satisfactory. Calculations for ice milling conditions indicated that resonant vibration will not be induced by milling ice at the above shaft speeds. The program allows the user to view the shaft torque of each shaft element in animated form to illustrate the distribution of shaft torque for a series of time steps. Figure 4.5 illustrates the shaft torque distribution at one point in time. This figure illustrates the decrease in the torsional amplitude through the main gear and the couplings.
Figure 4.5 Shaft Torque Distribution - Simulation Time = 0.103 Sec.
Comparison to Full Scale Data

Figure 4.6 and 4.7 illustrate time history plots of the shaft torque calculated using the Shaft Modeling Tool Kit model and that measured at full scale. As can be seen in these figures, there is good agreement between the results using the Shaft Modeling Tool Kit and the measured full scale data. The program has also been used to estimate the ice torque on propellers by comparing measured to calculated shaft torque time histories for various input torques on the propeller of the corresponding shaftline model.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Transportation Development Centre of Transport Canada, an Agency of the Canadian Federal Government located in Montreal, Quebec, for their financial support of the development of this program.

In addition the following personnel deserve special acknowledgement for their valuable assistance:

- Mr. Pierre L. Semery (TDC Project Officer)
- Arthur Karton (Computer Programmer for Fleet Technology Limited)
- Mr. Mike Smyth (A graduate student in Structural Engineering at the University of Calgary)
- Vlodec Laskow (Arctic Research and Development Ltd.)
- All the Fleet Technology Personnel involved in the development of this program
Figure 4.6 Comparison of Full Scale Shaft Torque with Model Results (Single Impact Event)
Figure 4.7 Comparison of Full Scale Shaft Torque with Model Results (Milling Event)
5.0. BIBLIOGRAPHY


Bathe, K.J., Wilson, E.L., Peterson, F.E., SAP IV - A Structural Analysis Program for Static and Dynamic Response of Linear System, Earthquake Engineering Research Centre Report No. 73 -11, University of California, Berkeley, California, 1974.


Kotras, T., Humphreys, D., Baird, A., Morris, G., Morley, G., Determination of Propeller-Ice Milling Loads


